The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2 ***Version 2.0***

Date: November 14, 2024 Course: EE 313 Evans

Name: ____
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **No AI tools allowed**. As mentioned on the course syllabus, you may not use GPT or other AI tools during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except the proctor for the test, and that you did not provide help, directly or indirectly, to another student taking this exam.

Problem	*Point Value*	*Your score*	*Topic*
1	27		System Properties
2	29		FIR Filter Analysis
3	24		System Identification
4	20		Filter Design
Total	100		

Problem 2.1. *System Properties*. 27 points.

Each discrete-time system has input $x[n]$ and output $y[n]$, and $x[n]$ and $y[n]$ might be complex-valued.

Determine if each system is linear or nonlinear, time-invariant or time-varying, and bounded-input bounded-output (BIBO) stable or unstable.

You must either prove that the system property holds in the case of linearity, time-invariance, or stability, or provide a counter-example that the property does not hold. Providing an answer without any justification will earn 0 points.

Part	***System Name***	***System Formula***	***Linear?***	***Time-Invariant?***	***BIBO Stable?***
(a)	Add a DC Offset	$y[n] = x[n] + 1$ for $-\infty < n < \infty$			
(b)	Reciprocal	$y[n] = \frac{1}{x[n]}$ for $-\infty < n < \infty$			
(c)	Scaling of the time axis	$y[n] = x[2\,n]$ for $-\infty < n < \infty$			

(a) Add a DC offset to the input signal: $y[n] = x[n] + 1$ for $-\infty < n < \infty$. *9 points*.

(b) Reciprocal: $y[n] = \frac{1}{x[n]}$ for $-\infty < n < \infty$

(c) Scaling of the time axis: $y[n] = x[2\,n]$ for $-\infty < n < \infty$ *9 points*.

Problem 2.2 *FIR Filter Analysis*. 29 points.

Consider a causal linear time-invariant (LTI) discrete-time finite impulse response (FIR) filter with input $x[n]$ and output $y[n]$ observed for $n \geq 0$ governed by

$$y[n] = x[n] + \ b\, x[n-1] + x[n-2] \quad \text{for } n \geq 0$$

where $b = -2\cos(\widehat{\omega}_0)$ and $\widehat{\omega}_0$ is a constant that is discrete-time frequency in units of rad/sample.

(a) What are the initial condition(s) and their value(s)? Why? *5 points*.

(b) Derive a formula for the transfer function in the z-domain including the region of convergence. *6 points*.

(c) Derive a formula for the discrete-time frequency response of the filter. Justify your approach. *6 points*.

(d) Which best describes the filter's magnitude response and why? Lowpass, highpass, bandpass, bandstop, notch/nulling, or allpass. *6 points*.

(e) Give all possible conditions on the constant b so that the FIR filter has constant group delay (i.e. linear phase). Compute the constant group delay. *6 points*.

Problem 2.3 *System Identification.* 24 points.

You are given several causal discrete-time linear time-invariant (LTI) systems each with unknown impulse response $h[n]$ but you are able to observe the input signal $x[n]$ and output signal $y[n]$ for $-\infty < n < \infty$.

For reference, the unit step function $u[n]$ is defined as

$$u[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$y[n]$	$Y(z)$	Region of Convergence
$\delta[n]$	1	all z
$\delta[n - n_0]$	z^{-n_0}	$z \neq 0$
$u[n]$	$\dfrac{1}{1 - z^{-1}}$	$\lvert z\rvert > 1$
$(n + 1)\, u[n]$	$\left(\dfrac{1}{1 - z^{-1}}\right)^2$	$\lvert z\rvert > 1$
$a^n\, u[n]$	$\dfrac{1}{1 - a\, z^{-1}}$	$\lvert z\rvert > \lvert a\rvert$

(a) When the input is $x[n] = \delta[n] - \delta[n - 1]$, the output is

$$y[n] = \delta[n] - \delta[n - 1] - \delta[n - 2] + \delta[n - 3]$$

i. Find the *finite* impulse response $h[n]$. *6 points*.

ii. Verify your answer by convolving $h[n]$ and $x[n]$. *6 points*.

(b) When the input is $x[n] = u[n]$, the output is $y[n] = (n + 1)\, u[n]$.

i. Find the *infinite* impulse response $h[n]$. *6 points*.

ii. Verify your answer by convolving $h[n]$ and $x[n]$. *6 points*.

Problem 2.4. *Filter Design.* 20 points.

Consider designing discrete-time linear time-invariant (LTI) infinite impulse response (IIR) filters.

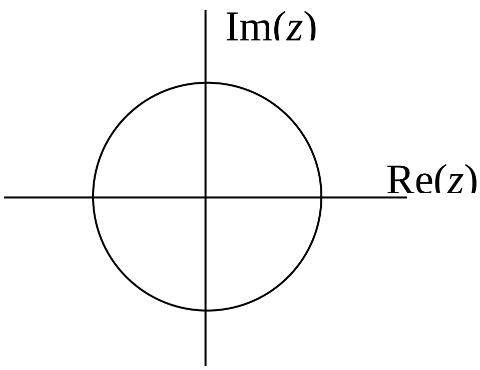
A second-order LTI IIR filter has zeros z_0 and z_1 and poles p_0 and p_1, and its transfer function in the z-domain (where C is a constant) is

$$H(z) = C\,\frac{(z - z_0)(z - z_1)}{(z - p_0)(z - p_1)}$$

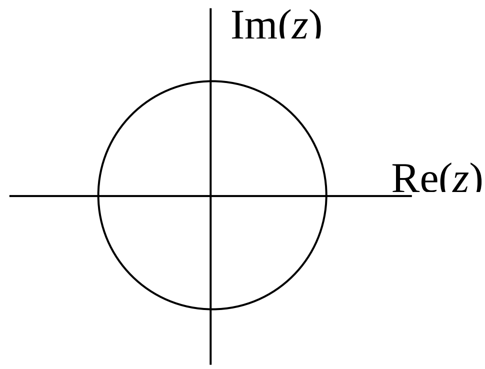
In this problem, **the poles and zeros will be complex-valued but <u>not real-valued</u>.** The imaginary part of the complex number cannot be zero, and the real part of the complex number can be anything.

Give numeric values for zeros z_0 and z_1 and poles p_0 and p_1 to give each magnitude response below, place the zeros and poles on the pole-zero diagram, and explain your reasoning. Please use O to indicate zero locations and X to indicate pole locations. **For each part, each zero and each pole must have a unique value. No two can have the same value.**

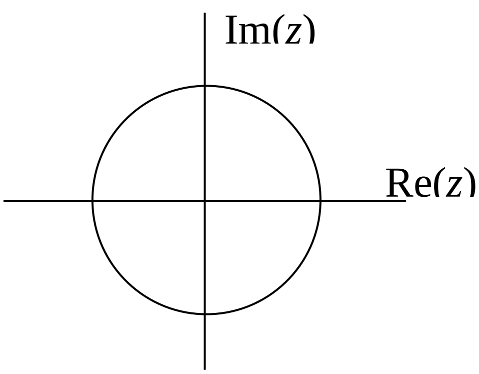
(a) Lowpass filter



(b) Highpass filter



(c) Allpass filter



(d) Notch filter